

The Observational Sorites Paradox

1. Preamble.

Think of the Sorites Paradox as a kind of disease.¹ Better still, since the Paradox comes in various different forms, think of there being a family of distinct but closely related Sorites disorders which each present their own set of characteristic symptoms.² How many forms of the malady are there? There is the Standard Sorites Paradox; its variant, the No Sharp Boundaries Paradox; the Phenomenal Sorites Paradox; the Identity Sorites Paradox; and its phenomenal variant, the Phenomenal Identity Sorites Paradox.³ Is this list complete? No. It's missing a somewhat neglected version of the pathology: *The Observational Sorites Paradox*.⁴

A preliminary goal in what follows is to properly articulate the Observational Sorites Paradox, together with a related paradox which I dub *The Observational Paradox*. From there, I distinguish the Observational Sorites from a paradox with which it is easily conflated, namely the Phenomenal Sorites Paradox. Next, I outline six treatments of the Observational Sorites, some familiar, others less so. These are: two versions of Subvaluation; two versions of Supervaluation; and, two versions of Epistemicism. The aim is not to provide a comprehensive evaluation of the respective virtues and vices of these treatments, but rather to uncover what they have in common. The main goal in what follows is to exploit such commonality to develop a completely different kind of solution to the Observational Sorites—a solution which deploys only minimal theoretical resources. The result is what may be dubbed *Minimalism*. As we shall see, minimal treatments of paradox can prove to be just as effective as any non-minimal treatment, but without many of the untoward side-effects.

¹ Here, and in what follows, I deploy a medical metaphor due to Tarski (1969) who remarked: “The appearance of an antinomy is for me a symptom of disease”; see also Chihara (1979). This metaphor is developed further in my *Knowledge: In Sickness and in Health*.

² Much like Carditis, Cancer, or Influenza (see §25 for more on this).

³ At a push, there is also the Forced March Sorites (Horgan 1984).

⁴ Dummett (1975) was the first to isolate the Observational Sorites Paradox.

2. The Symptoms of the Standard Sorites Paradox.

The Standard Sorites Paradox is the *non-specific* form of the malady and has four main symptoms.⁵ The first symptom is that the paradox utilises some vague predicate, e.g., “is yellow”.⁶ The second symptom is that there is some suitably long series of colour patches such that:

- (1) The first patch is yellow.
- (2) The last patch is not yellow.

The third symptom is that the vagueness of “is yellow” makes it highly plausible—at least initially—that adjacent patches should both be yellow if one of them is.⁷ So, where x' is the successor of x in the sorites series, we have what is termed *The Induction Step*:

- (3) For all x , if x is yellow then x' is yellow.

The fourth symptom is that classical logic is valid.⁸ In particular, the principle of Mathematical Induction.⁹ Using this principle, from (1) and (3) we can conclude:

- (4) All patches in the series are yellow.

⁵ Here, and below, I focus on predicate vagueness.

⁶ Other prototypical vague predicates are: “is tall”, “is a child”, “is bald”, “is a heap”, “is a tadpole”.

⁷ Sorites-susceptibility is one of the main symptoms of vagueness, see Greenough (2003).

⁸ For convenience, I take “classical logic” to mean first-order logic with identity (cf. Keefe 2000, p. 48).

⁹ One additional hallmark not considered here: the reasoning proceeds within a single context wherein the extension of the predicate remains fixed.

It follows that the last patch is yellow, which contradicts (2).¹⁰ Thus, from initially plausible premises, via initially plausible reasoning, we derive something implausible—a contradiction. Paradox.¹¹

3. The Symptoms of the Phenomenal Sorites Paradox.

The Phenomenal Sorites Paradox, in contrast, is a *specific* form of the pathology and has five main symptoms. The first is that it employs a phenomenal predicate, e.g. “looks yellow”.¹² The second is that the sorites series for “looks yellow” is a phenomenal continuum (in respect of colour) in the following sense:

- (1) The first patch looks yellow.
- (2) The last patch does not look yellow.
- (3) For all x , x is phenomenally identical to x' (in respect of colour).¹³

The third symptom is that a *strong* form of phenomenal identity is employed, whereby:

- (4) x is phenomenally identical to y (in respect of colour) if and only if x looks the same as y (in respect of colour).¹⁴

¹⁰ The Standard Sorites can also proceed from (1), modus ponens, and the conditionals: if the first member of the series is F the second is, if the second is, so is the third, ...

¹¹ The No Sharp Boundaries Paradox uses the premise: there is no x such that x is F and x' is not- F . Given (1) and (2), we derive a contradiction via either the classical or intuitionistic least number principle. So, classical logic is not an essential symptom.

¹² Further examples: “feels heavy”, “sounds high-pitched”, “tastes salty”, “smells burnt”, “appears yellow to me”, “looks like yellow”, “looks as if it is yellow”. More controversially: “looks not yellow”.

¹³ Here (strong) phenomenal identity, unlike identity, is relative. The absolute form is: x is phenomenally identical to y *in all respects*.

¹⁴ A weak phenomenal continuum employs *weak* phenomenal identity whereby neighbouring members do not look different (in respect of colour). Weak phenomenal identity entails strong phenomenal identity only given controversial assumptions. See Greenough “The Phenomenal Sorites Paradox”, ms2.

The fourth symptom is that the following principle is plausible:

(5) For all x , if x looks the same as x' (in respect of colour) then if x looks yellow then x' looks yellow.

The fifth symptom is that classical logic is valid. So, from (3) and (4), we can derive the Induction Step:

(5) For all x , if x looks yellow then x' looks yellow.

As above, we can conclude, via Mathematical Induction:

(6) All patches in the series look yellow.

This entails that the last patch looks yellow, which contradicts (2).¹⁵ Thus, from initially plausible premises, via initially plausible reasoning, we derive something highly implausible—a contradiction. Paradox.¹⁶

4. The Phenomenal Sorites and Standard Sorites Compared.

The Induction Step in the Standard Sorites enforces that vague predicates are tolerant—that they draw no boundary across their dimension of comparison. Why is this principle taken to be so initially plausible? The standard answers are: it is just highly intuitive; that it is absurd to think there is a cut-off in the series; that the very nature of vague predicates

¹⁵ Again, one could also employ a series of conditionals and successive applications of modus ponens. Equally, there is also a No-Sharp Boundaries variant of the Phenomenal Sorites.

¹⁶ The Phenomenal Identity Sorites Paradox uses the assumption that there is a strong phenomenal continuum for “looks yellow”; plus the abstraction principle: the look of x is the look of y if and only if x looks the same as y ; plus the classical assumption that identity is transitive.

means they are tolerant to marginal change across the series. These answers provide a *non-specific* motivation for the tolerance of vague terms—that’s why the Standard Sorites represents the non-specific version of the disorder.¹⁷

In contrast, the Phenomenal Sorites is a specific version of the sickness because it offers a specific motivation for taking a phenomenal predicate such as “looks yellow” to be tolerant, namely, that this phenomenal predicate ranges over a strong phenomenal continuum *and* that if adjacent members of this continuum look the same (in respect of colour) then if one looks yellow, then so does the other.

So, even though a presentation of the Paradox may, for example, employ a phenomenal predicate or an observational predicate, this does not show that we are dealing with the Phenomenal Sorites, or the Observational Sorites, respectively. Rather, to diagnose which particular version of the malady has taken hold, we also need to isolate the particular motivation given for the claim that the predicate employed is tolerant. Furthermore, the specific motivation for the tolerance of phenomenal predicates is more compelling, on the face of it, than the non-specific motivation given for the tolerance of ordinary vague predicates in the Standard Paradox. This explains why it is often thought that the Phenomenal Sorites is much harder to treat than the Standard Sorites.

5. Observational Predicates: The Traditional Conception.

On what may be termed *The Traditional Conception*, a predicate is observational when and only when a subject is in a position to know *via observation* whether or not the predicate applies—at least if the subject is functioning normally and conditions for observation are

¹⁷ Likewise for the No Sharp Boundaries Paradox.

also normal.¹⁸ Here observation is just a non-specific method of coming to know, which subsumes: looking, tasting, smelling, listening, feeling, and so on. Prototypical observational predicates include: “is yellow”, “is hot”, “is heavy”.¹⁹ Prototypical non-observational predicates include: “reflects wavelength of light n ”, “is 32 degrees in temperature”, “weighs 50KG”.²⁰

More rigorously, according to the Traditional Conception, say that a predicate “is F ” is observational just in case, under N-conditions, the following principles are valid:²¹

- (K1) Necessarily, for all x , if x is F then S knows (via observation) that x is F .
 (K2) Necessarily, for all x , if x is not- F then S knows (via observation) that x is not- F .²²

The N-Conditions are: (i) The subject S is in normal observational conditions—e.g., in the visual case, the object is in plain sight under normal lighting conditions. (ii) S is functioning normally—and so is not tired, drunk, delusional, hallucinating, and so on.²³

¹⁸ Hempel (1958, p. 42) says: “In regard to an observational term it is possible, under suitable circumstances, to decide whether the term does or does not apply to a given situation”. See also Carnap (1956), Dummett (1975), Wright (1975, 1976, 1987), Williamson (2013, pp. 43-44). For discussion of the observational/non-observational distinction see Dretske (1964). If observation is allowed to include introspection, then the Traditional Conception also counts various predicates concerning a subject’s occurrent mental states as observational. For example, “is painful”, “feels painful”, “feels hot”, and so on.

¹⁹ Also, the relational predicates: “is redder/hotter/heavier than”; the phenomenal predicates: “looks yellow”, “feels hot”, “feels heavy”; and, the phenomenal binary predicates “looks redder than”, “looks the same as”. All phenomenal predicates are observational, but not conversely since phenomenal predicates contain “appears”, “looks”, and cognate qualifiers.

²⁰ Carnap (1936/7, p. 455; 1966, p. 226) acknowledges that the difference between observational and non-observational terms is a matter of degree and relativised to an organism. Wright (1975, 1976) has a generous conception and includes, e.g., “is a heap”, on the grounds that we learn the meaning of this term through ostension.

²¹ Other versions of the Traditional Conception include: an observational, a doxastic, a justificationist, and a phenomenal conception, whereby “is F ” is observational just in case, under N-conditions, necessarily, if x is F /not- F then S observes/believes/ S is justified in believing (via observation)/it is looks to S that x is F /not- F .

²² (K1) and (K2) are here taken to record something stronger than a law-like or causal connection. That is controversial but nothing much depends on it below. For convenience, I have stated (K1) and (K2), and similar principles below, non-metalinguistically. Nothing hangs on this convenience for our purposes.

²³ As is widely acknowledged, it is difficult to specify normal conditions informatively, see Hardin (1983).

(iii) S is fully attending to the presentation of x . (iv) S is wondering whether x is F with the aim of forming a belief as to whether x is F . (v) S is using casual and unaided observation. (vi) S believes that the situation is normal such that they believe that they are in a position to observe whether x is F .²⁴

That there are observational predicates on the Traditional Conception has some initial plausibility. Under normal lighting conditions, there does not seem to be any barrier to prevent a normal subject from observing whether or not a particular presented colour patch is, for example, yellow. In other words, when dealing with observational properties, nothing is hidden from ordinary observation, under N-conditions.²⁵

6. The Symptoms of the Observational Sorites Paradox.

The Observational Sorites Paradox is also a specific version of the sickness. It, too, has five main symptoms. The first symptom is that:

(1) The N-Conditions obtain.

The second symptom is that the paradox invokes the Traditional Conception of an observational predicate, such as “is yellow”. Given (K1), (K2), and (1), we have:

(2) For all x , if x is yellow then a subject S knows (via observation) that x is yellow.

(3) For all x , if x is not-yellow then S knows (via observation) that x is not-yellow.

The third symptom is that this predicate ranges over what may be termed an *observational sorites series*, specified as follows:

²⁴ Further provisos are presumably required—such as some minimal rationality condition. I leave the specification of these to the defender of the Traditional Conception.

²⁵ For our purposes, we can take an observational property to be a property picked out by an observational predicate.

- (4) The first member of the series is yellow.
- (5) The last member is not yellow.
- (6) For all x , x is observationally indiscriminable from x' (in respect of colour).

The fourth symptom is that observational indiscriminability is given the epistemic reading whereby:²⁶

- (7) For all x , if x is observationally indiscriminable from x' (in respect of colour) then it cannot be the case that: S knows (via observation) that x is yellow and S knows (via observation) that x' is not yellow.²⁷

The fifth symptom is that classical logic is valid, and so from (6) and (7) we derive:

- (8) There cannot be an x such that: S knows (via observation) that x is yellow and S knows (via observation) that x' is not yellow.

Given (2) and (3), from (8) we can derive the No Sharp Boundaries Principle:

- (9) There is no x such that: x is yellow and x' is not yellow.

This entails:

- (10) For all x , if x is yellow then x' is yellow.

²⁶ Williamson (1990/2013) is the *locus classicus* for an epistemic conception of *numerical* indiscriminability: x is indiscriminable from y (for S) when and only when S is not able to activate the knowledge that x and y are distinct. I adapt this view to yield a qualitative version of observational indiscriminability.

²⁷ The modal “ably” co-varies with modal “can”. Likewise for similar principles below.

From (10), given Mathematical Induction, and (4), we derive that all patches in the series are yellow, which contradicts (5). Since the premises and reasoning are both initially plausible. Paradox.²⁸

7. The Observational Sorites and Phenomenal Sorites Compared.

It ought to be clear that the Observational Sorites and the Phenomenal Sorites are two very different paradoxes, as the following table shows:

	The Phenomenal Sorites for “looks yellow”.	The Observational Sorites for “is yellow”.
Type of predicate.	Phenomenal only.	Observational: either non-phenomenal or phenomenal.
N-Conditions Obtain	Not necessarily.	Yes.
Assumption One	The sorites series is a phenomenal continuum: neighbouring members are phenomenally identical (in respect of colour)	The sorites series is an observational sorites series: neighbouring members are observationally indiscriminable (in respect of colour).
Assumption Two	Strong phenomenal identity is employed: adjacent members look the same (in respect of colour).	Observational Indiscriminability is employed: there is no x such that: S knows (via observation) that x is yellow and S knows (via observation) that x’ is not yellow.
Assumption Three	If adjacent members look the same (in respect of colour) then if one looks yellow so does the other.	Traditional Conception of Observational Predicates: “is yellow” is observational: (a) If x is yellow then S knows (via observation) that x is yellow, (b) If x is not-yellow then S knows (via observation) that x is not-yellow.
So,	n/a	If there is no x such that S knows (via observation) that x is yellow and S knows (via observation) that x’ is not yellow then: if x is yellow then x’ is yellow.
So,	“looks yellow” is tolerant: If x looks yellow then x’ looks yellow.	“is yellow” is tolerant: If x is yellow then x’ is yellow.

²⁸ Cf. Dummett (1975), Wright (1975, 1976, 197), Williamson (2013, ch.3).

Despite these differences, the Phenomenal Sorites Paradox and the Observational Sorites are often conflated—or the latter is overlooked in favour of the former. There are various related reasons for this.

Often phenomenal predicates and observational predicates are taken to be much the same thing. Very often, *looking the same as* is either identified or taken to be co-extensive with *is not discriminably different from*. However, phenomenal identity is not the same relation as indiscriminability; it is not even the same relation as *phenomenal* indiscriminability. Indiscriminability is an epistemic relation, to be specified in terms of knowledge or some knowledge-like surrogate.²⁹ Phenomenal identity is a purely phenomenal relation, to be specified in terms of phenomenal notions.³⁰ Moreover, objects which are phenomenally identical, may still be discriminable, so the two relations are not even co-extensive.³¹ Perhaps the most you can get is: if N-conditions obtain then x does not look different from y if and only if x is not *phenomenally* indiscriminable from y—but note that here the former relation is the weak notion of phenomenal identity *does not look different from*, not the strong notion *looks the same as*.

8. What must an effective treatment of a paradox do?

²⁹ I am here setting aside statistical notions of indiscriminability, important as those notions are for for certain purposes.

³⁰ Dummett (1975) takes phenomenal predicates to be much the same thing as observational predicates. He, and Wright, also occasionally treat the relations *not observably different / not discriminably different from* as a being equivalent to *looks the same as*. Graff (2001) does so too in her abstract—but I take that to be a slip. Raffman (2000) treats perceptual indiscriminability to be equivalent to phenomenal identity. While de Clercq and Horsten (2004, ref.) say: “Indeed, it seems to us that the relevant notion of a phenomenal continuum [...] is to be understood in phenomenal terms from the start, e.g., in terms of ‘being perceptually indiscriminable’, or ‘looking the same’, or ‘looking homogeneous’, or in terms of ‘there being no appearance of an abrupt change’”. This is why I have laboured to properly distinguish the Observational Sorites Paradox from the Phenomenal Sorites Paradox.

³¹ See Greenough *Knowledge: In Sickness and in Health*, ms1, where this plays a crucial role in a minimalist treatment of Cartesian Scepticism.

It is a familiar thought that an effective treatment of a paradox must at least do the following two things:³²

(1) Provide good reason to: reject some premise(s); or, reject some (logical) rule of inference; or reject some presupposition(s) of the proof; or, “bite the bullet” and endorse the conclusion.³³

(2) Give an explanation as to why we were so susceptible to the paradox in the first place—despite the faults isolated in (1). Or, if biting the bullet, we must explain why the conclusion struck us as so implausible despite being acceptable after all.

One further—somewhat less discussed—desideratum is that any successful treatment should also:

(3) Obey a maxim of minimal mutilation: make any restrictions as conservative as possible. This maxim is typically applied to candidate revisions of classical logic (and classical semantics), but it ought to apply more broadly to any candidate revisions of the assumptions, presuppositions, and rules of inference used in the proof.³⁴

Finally, an effective treatment of some version of the Sorites Paradox must entail the following two basic symptoms of vagueness:³⁵

Minimal Symptom of Vagueness One: When x is borderline for “is F ”, a subject S does not know that x is F and does not know that x is not- F .

³² See, e.g., Chihara (1979).

³³ I assume an acceptable treatment cannot take all members of the series for “is F ” to be both F and not- F .

³⁴ Simons (1996, p. 326) says that we need to “rescue what can be rescued of classical logic”. See also Keefe (2000, p. 48), and Fine (1975, p. 286).

³⁵ In terms of the medical metaphor, any candidate theory of cancer which fails to explain the symptoms of cancer is a bad theory of cancer from the outset.

Minimal Symptom of Vagueness Two: Vague predicates are *epistemically* tolerant—they draw no known boundary across their associated dimension of comparison. That is, when “is *F*” is vague then there is no *x* such that: *S* knows that *x* is *F* and *S* knows that *x*’ is not-*F*.³⁶

We can think of these symptoms as representing material adequacy conditions on any acceptable, substantial theory of vagueness.³⁷

Before turning to various candidate treatments, there is one further paradox we must first consider.

9. The Observational Paradox.

Do we really need the Observational Sorites Paradox to put the Traditional Conception of observational predicates and/or classical logic under pressure given the phenomenon of vagueness? As it turns out: No. All we need is the three following assumptions:

(A1) The Minimal Symptom of Vagueness One: *S* does not know whether *x* is *F*, when *x* is borderline for “is *F*”.

(A2) Classical logic.

(A3) The Traditional Conception of observational predicates: “is *F*” is observational if and only if, under N-Conditions: For all *x*, if *S* knows that *x* is *F*/not-*F* then *S* knows (via observation) that *x* is *F*/not-*F*.

Given these three assumptions, it follows that *x* is not-*F* and it is not the case that *x* is not-*F*, when *x* is borderline for “is *F*”. Contradiction. Paradox. Call this *the Observational Paradox*.

³⁶ See Greenough (2003). Given classical logic, and some plausible assumptions, such as the monotonicity of the sorites series, these two symptoms of vagueness are inter-derivable.

³⁷ That is, the statement of these symptoms function in just the same way as Tarski’s material adequacy condition for any substantial theory of truth, namely, the T-schema: *S* is true if and only if *p* (where *S* says that *p*). The minimal theory of vagueness in Greenough (2003) was conceived as an analogue to Tarski’s minimal theory of truth (which is not to be confused with his semantic theory of truth).

The Observational Paradox is just as much a paradox as the Observational Sorites Paradox, though somewhat less exotic perhaps. That's because (A2) and (A3) are assumptions already employed in the Observational Sorites Paradox; and while (A1) is not explicitly used, it's a minimal symptom of vagueness, so is highly plausible. Indeed, given certain assumptions, (A1) is inter-derivable, given (A2), with the assumption that adjacent members of the series are observationally indiscriminable.³⁸ So, the two paradoxes effectively involve the same assumptions.³⁹ So, a successful remedy of the one should yield a successful remedy of the other. The key difference between the two paradoxes, of course, is that the Observational Paradox does not involve soritical reasoning—hence its truncated name.

What remedies, then, are available for these Observational Paradoxes? In the next few sections, I articulate and develop some familiar and less familiar treatments. The main goal will be to uncover what they have in common. In doing so, we shall also assess each remedy for material adequacy.⁴⁰

10. Non-Specific Subvaluation.

³⁸ See Greenough (2003).

³⁹ This does *not* make the two paradoxes equivalent however since the property of *being initially plausible (to degree n)* is not transmitted over plausible entailment.

⁴⁰ One prominent response not considered below is Incoherentism. When applied to the Observational Sorites, one rather radical version of this view runs as follows: the Paradox just reveals that the rules governing observational predicates are incoherent—they issue in contradictory instructions if sufficiently pressed (Dummett 1975). One such rule is a rule of tolerance: if x is observationally indiscriminable from y , and one has applied “is F ” to x on the basis of observation under N-Conditions, then one must also apply “is F ” to y . Indeed Dummett goes even further and argues that observational predicates are *ineradicably* incoherent such that there can be no logic for a language which contains observational predicates—not even some exotic non-classical logic. For one thing, such a view does not fare well with respect to the Observational Paradox just mooted. That's because there is no rule of tolerance at work underpinning the premises of this paradox. It would thus be rather perverse to respond to this paradox by throwing up one's hands and concluding that there is no logic of natural language. For another, Incoherentism—in the form expounded by Dummett—is a last-resort response which one should only invoke if all other treatments have proved ineffective. (And much like Chihara's response to Tarski's conceptual surgery for truth, one begins to wonder whether the cure is worse than the disease.)

A non-specific Subvaluational conception of vagueness has the following key features: extensionally vague sentences are both true and false; vagueness is a kind of multiple ambiguity; truth simpliciter is subtruth, namely truth in *some* acceptable sharpening; validity is necessary preservation of subtruth; the classical tautologies are all valid, and so the Law of Non-Contradiction is valid; however, modus ponens, &-I (adjunction), and Universal Generalisation are all invalid.⁴¹

Suppose we have a cartoon sorites series of three members whereby, on a subvaluational conception, "1 is *F*" is true only, "2 is *F*" is both true and false, and "3 is *F*" is false only. Given this, the two conditionals "If 1 is *F* then 2 is *F*", and "if 2 is *F* then 3 is *F*" are true on some sharpening (and false on some sharpening), and so true simpliciter (and false simpliciter). So, one form of tolerance intuition is preserved on this conception: these conditionals are true. Indeed, if one also assumes that warranted assertion is tied to truth, and not to non-falsity, then a subject is warranted in asserting these conditionals.⁴² However, there is no sharpening on which the conditionals are both true, and so the Universal Generalisation "For all *x*, if *x* is *F* then *x*' is *F*" is false only—that is, not true. This explains why the rule of Universal Generalisation fails, since this conditional is untrue despite all its instances being true.⁴³

So, the main premise of the Standard Sorites Paradox is untrue, which allows us to reject this premise and thus prevent the disease from taking hold. The diagnosis as to why we were so susceptible to the Standard Sorites then runs, I take it, roughly as follows: we have failed to realise that, when dealing with vagueness, just because each conditional claim about a sorites series is true does not mean that we can collect these conditionals such that they are all true together.

⁴¹ See Hyde (1997). Here, and in what follows, I shall ignore issues concerning higher-order vagueness, important as they are.

⁴² The negations of these tolerance conditionals are true (and false) and so their negations are warrantably assertible too. Thus, it remains very unclear in what sense tolerance intuitions are really preserved after all.

⁴³ When *x* is borderline for "is *F*" then "*x* is *F*" and "*x* is not-*F*" are both true and false and yet "*x* is *F* and not-*F*" is false on all sharpenings and so false only—that is, not true. This demonstrates why adjunction fails.

When the Standard Sorites Paradox is stated as a series of conditionals then while all the premises of the Paradox are true, modus ponens fails. Given our cartoon three member series, the conditional “If 2 is F then 3 is F ” is true on some (but not all) sharpenings, and so true, and “2 is F ” is also true (and false). However, “3 is F ” is untrue. And since validity is necessary preservation of truth simpliciter (subtruth) then modus ponens is invalid.⁴⁴

11. Type I Subvaluation.

To get clear about the subvaluational treatment of the Observational Sorites Paradox, we need to get clear on what such a view says about the two minimal symptoms of vagueness.⁴⁵ What may be termed, *Type I Subvaluation* entails that a subject is in a position to know that x is F and is in a position to know that x is not- F , when x is borderline for “is F ”. In particular, suppose a subject knows that x is borderline case and comes to know that a subvaluational semantics for vagueness is called for. By competent deduction, they can come to know that it is true that x is F and know that it is false that x is F . On the Type I version of the view, the following conditionals are true (and detachable): if S knows that it is true that x is F then S knows that x is F ; if S knows that it is false that x is F then S knows that x is not- F . So, a subject can come to know that x is F and know that x is not- F , when x is borderline for “is F ”. Immediate upshot: it is possible to know that x is F , even when it is false that x is F . That is, the following conditionals are not true: If S knows that x is F then it is not false that x is F ; if S knows that x is not- F then it is not true that x is F .

Moreover, such a view can preserve (K1) and (K2): for all x , if x is F /not- F then S knows (via observation) that x is F /not- F . When x is borderline for “is F ”, then x is F , x is not- F ; and given that these conditionals are detachable, one can use modus ponens to

⁴⁴ See Greenough, “The Phenomenal Sorites Paradox”, ms2, for the subvaluational response to the Phenomenal Sorites.

⁴⁵ Extant gluttony views vagueness—Priest (2003), Weber (2010), Ripley (2011)—are conspicuously silent as to whether or not knowledge is possible in the borderline area. Graham Priest (p.c.) tells me that there is not a developed dialethic view on the matter. There really needs to be, otherwise we cannot assess a dialethic conception of vagueness for material adequacy.

conclude that S knows that x is F and S knows that x is not- F , when x is borderline.⁴⁶ As ought to be clear, the first minimal symptom of vagueness is not entailed by such a conception. Indeed, the view entails the negation of this symptom. That provides an immediate response the Observational Paradox: the first assumption of this paradox, namely, the first Minimal Symptom of Vagueness, is rejected.

With respect to the second symptom of vagueness, and our cartoon sorites series, Type I Subvaluation entails that S knows that 1 is F , S knows that 2 is F , S knows that 2 is not- F , and S knows that 3 is not- F . So, there is an x such that: S knows that x is F and S knows that x' is not- F . So, vague predicates are not epistemically tolerant on such a conception. Indeed, on this view, vague predicates draw multiple known boundaries across the borderline area.⁴⁷ In effect, this entails that some adjacent members of the sorites series for “is yellow” are discriminable in respect of colour since there is some patch x such that one is able to know that x is yellow *and* know that its successor is not yellow! Hence, there are no observational sorites series on such a conception. Upshot: the Type I Subvaluational solution to the observational Sorites Paradox for “is yellow” is as follows: (i) “There is an x such that x is F and x' is not- F ” is true on all sharpenings, and so true only; (ii) the Traditional Conception of observational predicates remains valid; and, (iii) there are no observational sorites series since one can know, in the borderline area, that “is yellow” applies to x and know that this predicate fails to apply to its successor.

Type I Subvaluation is a complete non-starter—and not because of the radical thesis that extensionally vague sentences can be both true and false. But rather, because it fails to be materially adequate—it fails to entail the two minimal symptoms of vagueness.⁴⁸ Is there a materially adequate version of Subvaluation in the offing?

⁴⁶ The application of &I used here is valid since, higher-order vagueness aside, “S knows that x is F /not- F ” is true only or false only.

⁴⁷ What remains valid is that there is no x such that S knows that x is determinately F and S knows that x' is determinately not- F , where, on this conception, x is determinately F /not- F iff it is true only that x is F /not- F .

⁴⁸ And indeed fails to offer any kind of reason to think that the material adequacy conditions specified by a minimal theory of vagueness are not after all genuine symptoms of the condition.

12. Type II Subvaluation.

Given *Type II Subvaluation*, a subject does not (and indeed cannot) know whether x is F when x is borderline for “is F ”.⁴⁹ The following principles are thus true (and not false): If S knows that x is F then it is not false that x is F ; if S knows that x is not- F then it is not true that x is F . So, when it is both true and false that x is F , it immediately follows that one cannot know whether x is F . Even so, on such a conception, one is still in a position to know that it is true that x is F and in a position to know that it is false that x is F , when x is borderline. However, since truth/falsity is just truth/falsity in *some* acceptable sharpening, then the thought goes that knowledge that it is true/false that x is F is simply not sufficient to know whether x is F . (Analogy: knowing that a sentence “John is tall” is true on some disambiguation of “is tall” is not enough to know whether John is tall.)

Type II Subvaluation clearly entails the first symptom of vagueness, since one does not know whether x is F , when x is borderline. It also entails the second symptom of vagueness. Recall our cartoon sorites series. The following negated conjunctions are both true: “not: S knows that 1 is yellow and S knows that 2 is not yellow”, “not: S knows that 2 is yellow and S knows that 3 is not yellow”. So, there are no known cut-offs in the sorites series: vague predicates are epistemically tolerant.

What of the Traditional Conception of observational predicates? The following conditionals are both true and false, (and non-detachable): “If 2 is yellow then S knows (via observation) that 2 is yellow”, “If 2 is not-yellow then S knows (via observation) that 2 is not-yellow”. However, there is no sharpening in which these conditionals are both true and so the universal claim “For all x , if x is yellow/not-yellow then S knows (via observation) that x is yellow/not-yellow” is false only. Upshot: The Traditional Conception of observational predicates must be rejected.

The remedy for the Observational Sorites is clear: like Type I Subvaluation, there is no x such that x is yellow and x' is not-yellow. However, this does not entail that there are no observational sorites series, as on Type I Subvaluation, but rather that there are no

⁴⁹ A further option is that borderline sentences are both known to be true/false *and* not known to be true/false. That is an extremely far-fetched view which will not be considered here.

observational predicates as traditionally conceived. The result is, I submit, a far more plausible form of Subvaluation.

Furthermore, the following restricted versions of (K1) and (K2) remain valid on both Type I and Type II Subvaluation:

(KSUB1) Necessarily, for all x , if it is not false that x is F then S knows (via observation) that x is F .

(KSUB2) Necessarily, for all x , if it is not false that x is not- F then S knows (via observation) that x is not- F .

The thought then goes that we were seduced into thinking that (K1) and (K2) were valid because the Traditional Conception of observational properties was not simply not designed to take into account vagueness. When we do take into account vagueness (K1) and (K2) are to be replaced with (KSUB1) and (KSUB2). Can we reinstate the Observational Sorites Paradox using (KSUB1) and (KSUB2) and the existence of an observational sorites series? No. The most we can infer is the harmless: there is no x such that: it is not false that x is F and it is not false that x' is not- F .

Finally, with respect to the Observational Paradox, the Traditional Conception of Observational Predicates is, as we have seen, invalid on a Type II Subvaluational view, so premise (A3) of the Paradox is rejected. Can the paradox be reinstated using the following conditionals: "If 2 is yellow then S knows (via observation) that 2 is yellow", "If 2 is not yellow then S knows (via observation) that 2 is not-yellow"? Given our cartoon sorites series, both these conditionals are true (and false). Moreover, we can use modus tollens to conclude that 2 is not yellow and to conclude that 2 is yellow. However, we cannot use adjunction to further conclude that 2 is both yellow and not-yellow. So, no genuine contradiction.⁵⁰ Can we reinstate the Observational Paradox using (KSUB1) and (KSUB2)? No. The most that we can infer is that: it is false that x is F and it is false that x is not- F , when x is borderline for "is F ", but that is just what is to be expected on a Subvaluational conception.

⁵⁰ What also fails on a subvaluational semantics is a principle of substitution: one cannot substitute " x is F " for " S knows that x is F ". Mutatis mutandis for "It is true that p " / "It is true that not- p " and " p " / "not- p ", respectively, otherwise we could infer " p and not- p " from "It is true that p and it is true that not- p ".

13. Non-Specific Supervaluation.

A non-specific Supervaluational conception of vagueness has the following key features: extensionally vague sentences are neither true nor false; vagueness is a kind of (semantic) incompleteness; truth simpliciter is supertruth, namely truth in *all* acceptable sharpenings; validity is necessary preservation of supertruth; the classical tautologies are all valid, and so the Law of Excluded Middle is valid; however, modus tollens, Or-Elimination, and Existential-Elimination are all invalid.⁵¹

With respect to the Standard Sorites Paradox, consider now a supervaluational reading of our cartoon sorites series whereby “1 is *F*” is true, “2 is *F*” is neither true nor false, and “3 is *F*” is false. On a supervaluational view, The Induction Step is false in all sharpenings and so false. It follows that there is an *x* such that *x* is yellow and *x*’ is not yellow. However, while this existential claim is true, it has no true instance: it is true on all sharpenings, and so supertrue, and yet the instances “1 is *F* and 2 is not-*F*” and “2 is *F* and 3 is not-*F*” are true on some but not all sharpenings and are thus not true (and not false).⁵² Tolerance intuitions are preserved after a fashion because while there is a boundary, it is not true of each candidate sharpening that it marks the cut-off. It is as if the predicate is complete enough to determine that there is a boundary but not sufficiently complete to determine where the boundary falls. We are susceptible to the paradox because we mistakenly think that this latter kind of incompleteness entails that vague predicates are tolerant in the sense that there is no *x* such that *x* is *F* and *x*’ is not-*F*.

With respect to the first minimal symptom of vagueness, Supervaluation entails that a subject does not (and cannot) know whether *x* is *F*, when *x* is borderline, because to know that *x* is *F*/not-*F* requires that it is true/false that *x* is *F*. With respect to the second symptom of vagueness, a supervaluational semantics validates the following: there is no *x*

⁵¹ See Dummett (1975), Fine (1975), Keefe (2000). Again, I shall ignore issues concerning higher-order vagueness, important as they are.

⁵² This just parallels the fact that on a supervaluational semantics a disjunction (e.g. $p \vee \text{not-}p$) can be true on all sharpenings even though each disjunct is true on some but not all sharpenings.

such that it is true that x is F and it is true that x' is not- F . It follows, given that knowledge entails truth, that there is no x such that S knows that x is F and S knows that x' is not- F . So, Supervaluation entails the second symptom of vagueness. Thus, Supervaluation is a materially adequate conception of vagueness. What, then, of the Observational Paradoxes?

14. Type I Supervaluation.

Given Type I Supervaluation, the conditionals (K1) and (K2) have instances with an antecedent that is not true and a consequent that is false. On what is perhaps the most common conception of the conditional for a gappy logic, these instances are invalid as there is a “drop” in truth-value from not-true to false from antecedent to consequent.⁵³ So, Type I Supervaluation entails that there are no observational predicates as traditionally conceived. This provides an immediate resolution of the Observational Sorites since without (K1) and (K2) the paradox is blocked. Likewise, this blocks the Observational Paradox too since the Traditional Conception is given up.

Furthermore, the following restricted versions of (K1) and (K2) remain valid:

(KSUP1) Necessarily, for all x , if it is true that x is F then S knows (via observation) that x is F .

(KSUP2) Necessarily, for all x , if it is true that x is not- F then S knows (via observation) that x is not- F .

Just as with the Subvaluational conception above, the thought goes that we were seduced into thinking that (K1) and (K2) were valid because the traditional conception of observational properties was not simply not designed to take into account vagueness. When we do take into account vagueness, (K1) and (K2) are to be replaced with (KSUP1) and (KSUP2) on a supervaluational conception.

Do (KSUP1) and (KSUP2) reinstate the Observational Sorites Paradox? No, the most that can be inferred is the following harmless principle: there is no x such that it is true

⁵³ Likewise for the conditional in Kleene’s logic K3 and Lukasiewicz’s logic L3.

that x is F and it is true that x' is not- F . Do these principles reinstate the Observational Paradox? No. The most that can be inferred is: it is not true that x is F and it is not true that x is not- F , when x is borderline for “is F ”, which is exactly what is to be expected on this view.

15. Type II Supervaluation.

As it turns out, a rather better version of Supervaluation is in the offing which arguably has an improved semantics for the conditional whereby the material conditional is not true if its antecedent is true and its consequent not true; true otherwise. Upshot: (K1) and (K2) turn out to be valid after all. However, the contrapositions of these conditionals, namely:

(K3) Necessarily, for all x , if S does not know (via observation) that x is yellow then x is not yellow.

(K4) Necessarily, for all x , if S does not know (via observation) that x is not yellow then x is yellow.

are invalid: they have instances with a true antecedent and untrue consequent. Upshot: classical contraposition is invalid; modus tollens likewise fails.⁵⁴

This still blocks the Observational Sorites, however, because the inference from “there cannot be an x such that: S knows (via observation) that x is yellow and S knows (via observation) that x' is not yellow” to “there cannot be an x such that x is yellow and x' is not yellow” employs either (K3) and (K4) (plus modus ponens) or (K1) and (K2) plus modus tollens. Since modus tollens fails because contraposition fails, and (K3) and (K4) are invalid then one can block the Observational Sorites without giving up on (K1) and (K2), at least so long as these principles employ a non-contraposible conditional.

What this means is that we have to adjust what we originally meant by the Traditional Conception of observational predicates. Such a conception should now be seen

⁵⁴ Modus Tollens is also invalid; as is Reductio ad Absurdum. However, Conditional Proof and the Deduction Theorem are both preserved. The advantage of this conditional is that it allows that the T-schema (p is true if and only if p) is valid at least so long as it is expressed using a non-contraposible biconditional. Cf. Greenough (2009); cf. Frege (1903, p. 65).

as including not only (K1) and (K2) but also (K3) and (K4). With that adjustment in place, both Type I and Type II Supervaluation give up on the Traditional Conception of observational predicates. What is retained on both conceptions is: (KSUP1) and (KSUP2).

16. Non-Specific Epistemicism.

On a non-specific form of Epistemicism, extensionally vague sentences are either true or false but we do not know which; classical logic and classical semantics are both valid; there is a fact of the matter as to whether a borderline object is, e.g., yellow; and the source of ignorance in borderline cases is due to our limited powers of discrimination. Thus, epistemic theories of vagueness give a purely epistemological explanation as to why vague predicates draw no known boundary.⁵⁵

The Standard Sorites Paradox is blocked because the Induction Step is false—the validity of classical logic ensures that. Why then did we mistakenly think that vague predicates are tolerant? One explanation is that we are in the grip of a residual verificationism whereby we (mistakenly) think that if we can't verify the whereabouts of a boundary, there must be none—and so vague predicates are tolerant. A related diagnosis is that we are in the grip of a Cartesian conception of understanding such that if one's knows the meaning of a vague sentence such as “John is tall”, and one knows the height of John then one is always in a position to know whether or not John is tall, which entails being in a position to know the boundary between tallness and non-tallness. Since we are not in such a position then there can be no such boundary. The thought then goes that once we free ourselves from Verificationism and such Cartesianism then the impetus to take vague predicates to be tolerant is considerably mitigated.⁵⁶

⁵⁵ This is why Sorensen's Truthmaker Gap conception of vagueness is not a form of Epistemicism despite being advertised as such (see Sorensen 2001, Greenough 2008).

⁵⁶ Both kinds of diagnosis can be found in Williamson (1994, p. xi, and pp. 209-212, respectively). In Greenough (2003), I alleged that the Standard Sorites is seductive because of a confusion of epistemic tolerance (no known boundary) with tolerance (no boundary).

The non-specific epistemicist treatment of the Observational Sorites runs as follows: classical logic is valid and so “is yellow” is intolerant; there are observational sorites series; however, in consequence, there are no observational predicates as traditionally conceived. The non-specific response to the Observational Paradox is also immediate: assumption (A3) is given up. Just as with Subvaluation and Supervaluation, the thought goes that we were seduced into thinking that (K1)-(K4) were valid because the Traditional Conception of observational properties was simply not designed to take into account vagueness. (We’ll get to the replacement weaker principles in a moment.)

Why exactly are we ignorant in the borderline area? The non-specific explanation goes as follows: human beings are not absolutely reliable detectors of the truth—their powers of discrimination are not perfect. That is, we cannot always discriminate (via observation) objects which are qualitatively different but relevantly similar. As it turns out, there are two conceptions of observational indiscriminability, which each give rise to a different form of Epistemicism. These more specific versions of Epistemicism each yield more specific explanations for the two minimal symptoms of vagueness.

17. Strong Epistemicism.

Given *Strong Epistemicism*, observational indiscriminability (in respect of colour), gets the strong reading whereby x is observationally indiscriminable from y (in respect of colour) if and only if: if x has colour C then, for all S can know via (observation), y has colour C too.⁵⁷ This strong conception of indiscriminability entails the following Margin for Error Principle:

(ME) For all x , if S knows (via observation) that x has colour C then x' has colour C too.

⁵⁷ Here the modality embedded in “indiscriminable” co-varies with the modality “can” on the right-hand-side of the biconditional.

ME entails that there are no known cut-offs. Suppose, for reductio, that S knows that: x is yellow and x' is not yellow. Given this, S knows that x is yellow. Given ME, it follows that x' is yellow too. But given the original assumption, S knows that x' is not yellow, from which it follows that x' is not yellow. Contradiction. So, Strong Epistemicism entails the second minimal symptom of vagueness.⁵⁸

21. Weak Epistemicism.

Given *Weak Epistemicism*, observational indiscriminability (in respect of colour) gets the weak reading whereby x is indiscriminable from y (in respect of colour) if and only if: if S knows that x has colour C then, for all S can know via (observation), y has colour C too. That is, there can be no x such that: S knows (via observation) that x has C and S knows (via observation) that x' lacks C. This weak conception of observational indiscriminability entails the following Minimal Margin for Error Principle:⁵⁹

(MME) For all x , if S knows (via observation) that x has colour C then S does not know that x' lacks colour C.⁶⁰

Since MME effectively expresses the claim that there are no known cut-offs, then it automatically entails the second symptom of vagueness.⁶¹

Finally, we can now formulate restricted versions of (K1) and (K2) which are available to both strong and weak forms of Epistemicism. These are:

⁵⁸ See Greenough (2003) for how the second symptom vagueness entails the first.

⁵⁹ ME entails MME given the factivity of "S knows that".

⁶⁰ In Greenough (2012), I use MME to undermine various Cartesian conception of the mental.

⁶¹ Again, see Greenough (2003) for why the second symptom entails the first.

(KEP1) Necessarily, for all x , if x is non-marginally F then S knows (via observation) that x is F .⁶²

(KEP2) Necessarily, for all x , if x is non-marginally not- F then S knows (via observation) that x is not- F .

Very roughly, x is non-marginally F if and only if x is not close to being not- F .⁶³ Do these principles reinstate the Observational Sorites Paradox? No. The most one can infer is: there is no x such that x is non-marginally F and x' is non-marginally not- F .⁶⁴ Do they reinstate the Observational Paradox? No. The most that one can infer is that x is marginally F and x is marginally not- F , when x is borderline for is “is F ”, which is just what should be expected on such a view.⁶⁵

22. Observational Predicates: The Minimal Conception.

Type I Subvaluation, as we saw above, is a complete non-starter because it fails to be materially adequate. All the other views passed the test for material adequacy. These remaining five treatments share a common diagnosis as to why we are (initially) susceptible to the Observational (Sorites) Paradox: the Traditional Conception of observational predicates—as specified by (K1)-(K4)—was simply not designed to take into account vagueness. When we do take into account vagueness, the Traditional Conception needs replacing. As we have seen, Type II Subvaluation replaces the Traditional Conception with (KSUB1) and (KSUB2); both types of Supervaluation replace this conception with (KSUP1) and (KSUP2); and, both types of Epistemicism replace this conception with (KEP1) and (KEP2).

Given this, a non-specific, minimal conception of observational predicates is available which is entailed by all these more substantive weaker conceptions:

⁶² Williamson (1994, p.183) effectively suggests just these principles.

⁶³ Here I employ the distal conception of closeness and not the “easy possibility” sense.

⁶⁴ That is, there is no x such that x is not close to being not- F and x' is not close to being F .

⁶⁵ That is, x is both close to being F and x is close to being not- F .

(KM1) Necessarily, for all x , if x is *determinately* F then S knows (via observation) that x is F .

(KM2) Necessarily, for all x , if x is *determinately* not- F then S knows (via observation) that x is not- F .

Here, x is *determinately* F if and only if: x is F and x is not borderline for F ; x is *determinately* not- F if and only if: x is not- F and x is not borderline for F .

Do these principles reinstate the paradoxes? As should be clear: No. With respect to the Observational Sorites, the most that can be inferred is: there is no x such that x is *determinately* F and x' is *determinately* not- F .⁶⁶ With respect to the Observational Paradox, the most that can be inferred is: there is an x such that x is not *determinately* F and x is not *determinately* not- F .⁶⁷

But now it looks like what may be termed a *Minimal Treatment* of the Observational Sorites Paradox is in the offing.

23. Minimalism about Vagueness

A minimal theory of vagueness is a theory which endeavours to theorise about vagueness from a perspective which is as neutral as possible on matters logical and philosophical. One of the main jobs of such a theory is to specify the symptomatology of vagueness; that is, provide an account of the characteristic symptoms of vagueness, together with the characteristic symptoms of the various kinds of sorites paradox. A deep, substantive theory of vagueness, in contrast, is a theory of the underlying nature of the disease which gives rise to these characteristic symptoms.

Despite not being the sort of theory to tell us very much about the nature of the underlying condition, a minimal theory nonetheless has several virtues. In particular, it

⁶⁶ That is, there is no x such that (x is F and is not borderline for F) and (x' is not- F and is not borderline for not- F).

⁶⁷ That is, there is an x such that it is not the case that (x is F and x is not borderline for F) and it is not the case that (x is not- F and x is not borderline for not- F).

can be used to identify the presence of vagueness via the presence of the characteristic symptoms; it can be used to distinguish vagueness from various other kinds of phenomena—such as semantic incompleteness—with which it may easily be confused; it can help ensure a mutually agreeable starting place in a debate which is fraught with disagreement and controversy; finally, as have seen, should some substantive theory of vagueness fail to entail the minimal symptoms of vagueness, and thus fail to be materially adequate, then we may dismiss its credentials from the outset.⁶⁸

Can a minimal theory of vagueness also furnish us with a lasting cure of the Sorites paradox, in any of its various forms? It might seem obvious that the answer has to be: No. Only a substantive theory of vagueness is equipped to do that, so the thought goes.⁶⁹ However, if we allow ourselves to be guided by the medical metaphor with which we began, then we will find that modern medicine is replete with successful treatments for diseases which are only partly, and sometimes hardly, understood.⁷⁰ So do we really need to have a good idea as to what what vagueness really is in order to treat the various paradoxes of vagueness? I submit that the answer is: No. Oftentimes, we can successfully address a philosophical puzzle without having a very good idea, or even much of an idea, of the underlying nature of the malady. Is there such a minimal treatment of the Observational Sorites Paradox and the Observational Paradox in the offing?

24. A Minimalist Treatment of the Observational (Sorites) Paradox.

Minimalism, conceived as a treatment of the Observational Sorites Paradox and the Observational Paradox, has the following key features: (1) The Traditional Conception of observational predicates is rejected. (2) This blocks both forms of the paradox. (3) The

⁶⁸ See Greenough (2003).

⁶⁹ I thought as much in Greenough (2003), but now I think that judgment was over-hasty as it overlooked the possibility of minimal treatments.

⁷⁰ One recent case is Ebola. This disease has been successfully treated with certain biologic agents—in particular a certain kind of antibody—even though the medical community does not know why this particular kind of antibody yields a successful treatment. {refs.]

Traditional Conception is replaced with the Minimal Conception of Observational predicates as given by (KM1) and (KM2) above. (4) These principles are sufficiently weak so as not to reinstate either paradox. (5) The resultant theory is neutral as to whether or not classical logic and classical semantics are valid in the presence of vagueness. (6) Indeed, the theory is neutral as to whether the nature of vagueness reflects our limited powers of discrimination or whether it results from some feature of language or some feature of the world. (7) Minimalism thus does not deploy any deep, substantial theory of vagueness in treating the condition. (8) The reason why the Traditional Conception is rejected is that while we may not fully comprehend the underlying nature of vagueness, we *do* know that the Traditional Conception is misconceived since this conception is given up on all materially adequate, substantial theories of vagueness. (9) The diagnosis as to why we were so susceptible to the Observational Paradoxes is that the Traditional Conception is misconceived: it was not designed to take into account the phenomenon of vagueness—all substantial conceptions agree on that so we can take it for granted too. (10) Even though the Traditional Conception must be rejected, there are still such things as observational predicates and there is still a workable distinction between observational and non-observational language. (11) Minimalism is materially adequate by default since the central feature of the theory is a specification of the two basic symptoms of vagueness.

Recall what a successful treatment of vagueness is supposed to do. It must: reject some feature of the paradoxical proof (or endorse the conclusion); provide reasons for such a rejection (or provide reasons to endorse the conclusion); diagnose why we were so susceptible to the paradox in the first place; obey a maxim of minimal mutilation: save what can be saved of the principles used in the paradox; and be materially adequate. The Minimal Treatment just outlined has all these features so it counts as a genuine candidate solution.⁷¹ Indeed, it has one feature which makes it arguably a better theory than any substantial theory: all partisans to the debate can help themselves to the theoretically lightweight cure on offer. Put another way: you don't need to prescribe an expensive medication when a cheap one will do the job just as well (if not better).

⁷¹ Though see below for a wrinkle with respect to the Maxim of Minimal Mutilation.

25. Specific and non-Specific Treatments.

At various points in the discussion so far, I have invoked the medical distinction between specific and non-specific diseases. An example of a non-specific condition is Carditis— inflammation of the heart.⁷² A more specific condition is Bacterial Carditis— inflammation of the heart due to a bacterial infection. A more specific condition still is Bacterial Pericarditis— inflammation of the sac surrounding the heart due to a bacterial infection. A more specific form again may involve a particular type of bacteria. Given the medical metaphor with which we began, we have also seen how there is a non-specific form of the Sorites Paradox—The Standard Sorites—and two more specific forms: the Observational Sorites and the Phenomenal Sorites. Indeed, there are arguably various more specific forms of the Phenomenal Sorites Paradox too.⁷³

In addition to the distinction between specific vs non-specific diseases, there is another important medical distinction between specific and non-specific treatments (and medications). A non-specific treatment can effectively treat (or is at least designed to treat) more than one version of a malady, or indeed various different maladies. An example would be penicillin which is a broad-spectrum antibiotic which is able to treat a variety of bacterial infections, including some forms of Bacterial Pericarditis. A specific treatment, meanwhile, is effective (or is at least designed to be effective) only against specific forms of a sickness. An example would be Vancomycin which is a narrow-spectrum antibiotic which can be used against, for example, Enterococcal Pericarditis, but is ineffective against many other forms of Bacterial Pericarditis, and is, of course, completely ineffective against fungal or viral forms of Pericarditis.

Again, given our medical metaphor, there are some philosophical treatments which are non-specific and which can be used to address not only the various different forms of, for example, the Sorites Paradox, but which may also be used against various other

⁷² Some medical practitioners doubt that this is a real condition at all on the grounds that it is a mere disjunctive condition: either Pericarditis or Myocarditis or Endocarditis. These disjuncts, in contrast, do represent natural kinds.

⁷³ See Greenough “The Phenomenal Sorites Paradox”, ms2.

puzzles and paradoxes too. One such treatment is Supervaluation.⁷⁴ This can be used to address: the sorites paradox, the liar paradox, the truth-teller, the problem of the many, the Ship of Theseus, the open future, and so on. Of course, various specific adjustments have to be made in order to make Supervaluation fully effective across such a wide range of paradoxes, but nonetheless the basic ingredient of the medication remains the same in each case.

A specific philosophical treatment is only effective against a specific form of a paradox, and will be ineffective against other forms, as well as ineffective against other kinds of paradox too.⁷⁵ For example, there is a specific (and minimal) treatment of one version of the Phenomenal Sorites Paradox which is completely ineffective against other forms of the Phenomenal Sorites, the Standard Sorites, and indeed any other paradox.⁷⁶ Likewise, Minimalism is capable of addressing The Observational Sorites and the Observational Paradox but is ineffective against all other paradoxes.⁷⁷

It's tempting to think that non-specific treatments should always be preferred over specific ones because non-specific treatments offer a uniform remedy to all the main forms of a disease—they yield a kind of local panacea. If only specific treatments were available then one will only be able to treat and eradicate a disease piecemeal. The thought then goes that uniformity and simplicity are to be preferred over plurality and complexity. Though commonplace, such an assumption is far too hasty. For one thing, it cannot be assumed from the outset that the various forms of the some paradox such as the Sorites are sufficiently uniform to call for a single, non-specific treatment.⁷⁸ Indeed, if we look to

⁷⁴ Other non-specific treatments are: Contextualism, Relativism, Epistemicism, Subvaluation, and more.

⁷⁵ There are, of course, degrees of specificity.

⁷⁶ Case in point: If “looks yellow” is given a strict occurrent reading whereby an object can only look yellow when a subject is looking at it, then a solution to an occurrent form of the Phenomenal Sorites is easy to come by: “looks yellow” turns out to have a known cut-off and is not sorites-susceptible. Such a solution is also pretty minimal as it deploys very little theory concerning the nature of vagueness.

⁷⁷ Not all specific treatments are minimal. Indeed, not all minimal treatments are specific.

⁷⁸ Wright (1987), in discussing his *Tachometer Paradox*, briefly floats the idea that different kinds of sorites paradox may well require different kinds of solution.

contemporary medicine for illumination then the picture is mixed. Sometimes non-specific treatments are preferred—such as a single flu vaccine which is able to prevent the spread of all flu viruses. Very often, however, specific treatments are preferred because they can be more effective and have fewer side effects. For example, some non-specific treatments of Bacterial Pericarditis use antibiotics which also kill the good bacteria in the gut—an undesirable side effect. More specific treatments do not have this result. The lesson here is that we should not merely be open to specific treatments in philosophy, we should be actively seeking them out since simplicity and uniformity often get to be outweighed by the increased effectiveness and/or the reduction in side effects that specific treatments may bring.

26. Objections and Replies.

Minimalist treatments for philosophical paradoxes will doubtless strike some as, at best, unhelpful, at worst, perverse. Let me try to assuage these worries somewhat by replying to some natural objections to the minimalist methodology on offer.

Objection One: You say that Minimalism is a specific treatment, and yet it is surely non-specific in that it deploys a non-specific, minimal conception of observational predicates.

Reply: There are two notions of specificity in play here. What may be termed *Treatment Specificity* is the kind that is determined by the range of diseases addressed by some treatment. What may be termed *Medication Specificity* concerns whether a single medication has a non-specific nature, such as the Minimal Conception of observational predicates, or a specific nature such as the Supervaluational Conception of observational predicates.

Objection Two: Even if the minimalist treatment is effective, the minimalist methodology on offer is too limited. To be sufficiently interesting, we would need minimal solutions of the

Standard Sorites Paradox, the Phenomenal Sorites Paradox, and indeed a range of minimal solutions of other long-standing paradoxes.

Reply: Agreed. As it turns out there is a minimal treatment of Cartesian Scepticism and Meta-Scepticism.⁷⁹ There is also a minimal treatment of the Standard Sorites in the offing. The semantic Paradoxes are also amenable to minimal treatments too.⁸⁰

Objection Three: The view on offer is just a deflationary, quietist view of philosophical theory. It doesn't offer genuine solutions to philosophical problems at all.

Reply: On one important reading, deflationism about truth is the view that the only thing to be said about truth is a set of minimal, theoretically lightweight claims. In terms of our medical metaphor, this amounts to the claim that the symptomatology of truth exhausts all there is to say about truth. Likewise deflationism about vagueness is the view that all there is to be said about vagueness is exhausted by a minimal theory of vagueness: there is nothing more to nature of vagueness than its symptoms (and nothing more to the theory of vagueness than a theory of its symptoms). Correspondingly, a deflationary philosophical methodology can be seen as the view that the only kind of effective solutions to philosophical problems are minimal, theoretically lightweight treatments.⁸¹

A non-deflationary theory of truth, meanwhile, says that there is more to say about truth than a set of minimal, theoretically lightweight claims: there is more to truth than its surface symptoms. Likewise, a non-deflationary view of vagueness, says there is more to vagueness than its surface symptoms: there is an underlying condition which gives rise to these symptoms. Correspondingly, a non-deflationary account of philosophical methodology says that there are not just minimal treatments of philosophical problems,

⁷⁹ See my *Knowledge: In Sickness and in Health*, ms1.

⁸⁰ Alas, there is not space to make good on this claim here.

⁸¹ Horwich (2011) defends a broadly deflationary approach to paradox. See also his (2012), and his deflationary theory of vagueness in his (1990). It's worth noting that Horwich maintains a commitment to classical logic and classical semantics. In contrast, the view immediately on offer is neutral as to the correct logical theory. See below for more discussion on this.

there are non-minimal treatments too, treatments which deploy substantial (and controversial) theoretical claims. So, while the minimal methodology on offer is compatible with a deflationary account of best philosophical procedure it does not require such a view. Indeed, the view I favour is the non-deflationary view which allows for both minimal and non-minimal treatments.⁸²

Objection Four: But then on your non-deflationary version of Minimalism, a minimal treatment of the Observational Sorites (or indeed of any other form of Sorites Paradox) is only of interest under the assumption that we do not know very much about the underlying nature of vagueness—hence why we should be neutral as to its nature. But that assumption of ignorance has not been argued for. Indeed, it is a highly controversial, and indeed sceptical assumption—one which should play no part in a lightweight, theory-neutral account of vagueness. Moreover, such an assumption is just false: we do know a lot about the nature of vagueness

Reply: It's true that minimal treatments take on a particular attraction when philosophical debates have become bogged down in squabbles, when there is little hope of philosophical consensus, when the prospect of gaining philosophical knowledge in some domain seems remote, if not quixotic. So, just as minimal treatments are attractive to the deflationist, they may also prove to be attractive as a response to various forms of scepticism concerning (substantial) philosophical knowledge. But the methodology on offer does not require such scepticism, just as it does not require a deflationary view of philosophy either. Indeed, the interest of Minimalism is dependent on either (or both) of two assumptions: that often substantial philosophical knowledge of vagueness is just hard to come by; that knowledge is not always needed for effective treatments. It does not depend on any sceptical assumption that we currently have very little philosophical knowledge or that it is impossible to achieve.

⁸² An inflationary view, in contrast, is the view which says that there are only non-minimal treatments to philosophical problems. This is the view which both Horwich and I resist, though Horwich (2012) is motivated by a broadly Wittgensteinian methodology which diagnoses paradox as issuing from a standing tendency for philosophical principles to be formulated with too much generality. In contrast, my own view is that paradox has many different sources.

Objection Five: But now you must allow a kind of pluralism regarding philosophical treatments: minimal treatments and non-minimal treatments.

Reply: That's right. Minimal treatments can co-exist with non-minimal treatments—the latter are (typically) just more specific versions of the former. Sometimes the latter will offer an improved treatment, but not always.⁸³

Objection Six: Minimalism is just a way of offering a disjunctive treatment, that is, a disjunction of non-minimal treatments. If so, then it is no real treatment at all. It would be akin to achieving theoretical neutrality by saying: the Observational Sorites is treatable via Subvaluation or Supervaluation or Epistemicism or ...

Reply: Recall the Minimal Conception of observational predicates whereby a predicate is observational just in case, under N-Conditions, the following principles are valid:
(KM1 / 2): For all x , if x is determinately F / not- F then x is known (via observation) to be F / not- F . If one treats the condition "is determinately F / not- F " as a place-holder notion which gets filled out differently according to each substantial theory of vagueness then the Minimal Conception of observational predicates, and the thus the minimal treatment of the Observational (Sorites) Paradox, would indeed be disjunctive in nature and thus of little interest. However, also recall that " x is determinately F " was taken to be equivalent to " x is F and is not a borderline case of F -ness". The notion of borderline case here is the one we effectively learn via ostension (and not via any substantial theory). So, there is a robust, non-disjunctive, theoretically lightweight notion of borderline case which can be exploited to give the Minimal Conception of observational predicates.⁸⁴

⁸³ In Greenough *Knowledge: In Sickness and in Health*, I argue that a minimal medication is all that is needed to combat Cartesian Scepticism. Substantial theories of knowledge either fail to be materially adequate (in the relevant sense) or add little to the effectiveness of the minimal medication.

⁸⁴ See Williamson (1994, p. 2).

Objection Seven: Minimalism is only effective if it has already been established that all substantial, materially adequate theories give up the Traditional Conception of observational predicates and replace it with a weaker conception which entails the Minimal Conception of observational predicates. Since this has not been established, then Minimalism is unmotivated.

Reply: Agreed this has not been established explicitly—that would involve a much longer paper to do the job in full. However, on all of the following (materially adequate) theories of vagueness, the Traditional Conception of observational predicates is replaceable with a weaker conception which entails the Minimal Conception: (i) Truth-functional truth-value glut conceptions.⁸⁵ (ii) Truth-functional truth-value gap conceptions.⁸⁶ (iii) Many-valued conceptions.⁸⁷ (iv) Intuitionist conceptions.⁸⁸ (v) Non-standard Supervaluation.⁸⁹ (vi)

⁸⁵ Priest (2003), Ripley (2011), Weber (2010).

⁸⁶ Hallden (1955), Korner (1966), Tye (1994), Hyde (2008).

⁸⁷ Goguen (1969), Lakoff (1973), Machina (1976), Peacocke (1981), Smith (2009). See also Edgington (1992) and Cook (2002).

⁸⁸ See Putnam (1983) and Wright (2001). Wright's Quandary view is an exotic mixture of intuitionistic logic and the thesis that "permissible disagreement is of the essence of vagueness". For criticism of this cocktail of medications, see Greenough (2009).

⁸⁹ McGee and McLaughlin (1995). On such a view, determinate truth is truth in all admissible sharpening, but truth simpliciter is not determinate truth (as on the standard supervaluational view of Fine (1975) and Keefe (2000)). Classical logic is fully preserved, and there is (irremediable) ignorance in the borderline area. The response to the Observational Paradox is immediate: give up the Traditional Conception of observational predicates.

Truthmaker Gap conceptions.⁹⁰ (vii) Moderate Incoherentism.⁹¹ (viii) Interest-Relative conceptions.⁹² (ix) Contextualism.⁹³

Objection Eight: You are conveniently forgetting *Tolerance Logics* which can fully respect the tolerance of vague predicates but block all forms of the Sorites by taking the consequence relation to be non-transitive. On such views, one cannot chain together all the steps of modus ponens employed in the series-of-conditionals formulation of the Standard Sorites Paradox.⁹⁴ Moreover, one can preserve the Traditional Conception of observational predicates on such view (so goes the thought).⁹⁵

⁹⁰ Sorensen (2001), cf. Greenough (2008). On such view, there is (irremediable) ignorance in the borderline area and classical logic is valid. The response to the Observational Paradox is immediate: give up the Traditional Conception of observational predicates.

⁹¹ Eklund (2001). Eklund does not explicitly address the Observational Sorites, but it is pretty clear how the view would go: vague predicates are (despite appearances) intolerant; however, our competence with vague expressions demands that we are at least (in some sense) disposed to accept that vague predicates are tolerant—which explains why we are so susceptible to the Sorites Paradox; it will also be part of our competence with observational predicates that our competence is (somehow) governed by the Traditional Conception; however this conception is invalid.

⁹² Graff-Fara (2000). On this view, vague predicates draw sharp cut-offs, but the extension of the predicate can shift as a function of shifting interests—and in particular, it can never be where we are looking/attending because interests are (almost) always such that two adjacent objects considered together as a pair are not category different. Hence, we can never locate the cut-off. So, there is no known boundary across the series. Given classical logic, there is always (some) ignorance in the borderline area. And so, the Traditional Conception of observational predicates must be given up. Outside of the borderline area, the Traditional Conception works fine, and so such a view entails the Minimal Conception of observational predicates.

⁹³ See Raffman (1994), Shapiro (2005). Even on a Contextualist view, the Traditional Conception does not hold because a fully rational subject can simply fail to form a belief, under N-Conditions, as to whether or not x is F in the borderline area. That's because it may strike that subject as *vague* whether or not x is F and so they refrain from forming a belief either way. See Greenough (2005) on "the problem of silence".

⁹⁴ Zardini (2008, 2013) was the first to resurrect this kind of approach in the contemporary debate. Dummett mentions, but immediately dismisses, such a view in his (1975). See also Cobreros, Égre, Ripley, and van Rooij (2012).

⁹⁵ Indeed, the proof given in Greenough (2003) which shows that having no known boundary entails ignorance in the borderline area is invalid on the logic proposed by Zardini (2008). Thanks to Elia for impressing this upon me. This fact does not change the main point in the text however: Tolerance Logics cannot escape giving a borderline conception—nor should they if such logics are to be materially adequate.

Reply: There is not space for a full answer here, but those who sponsor such non-transitive logics appear in the grip of the view that all conceptions which permit vagueness to be conceived in terms of being borderline are misconceiving the phenomenon: vagueness must *only* be captured in terms of tolerance. But some kind of borderline conception is unavoidable. Even if we reject the transitivity of the consequence relation, we will still be committed to a borderline area—such an area will just be the zone over which classical transitivity may break down. That such a pattern of reasoning does break down must be in virtue of some property possessed by one or more of the propositions involved in such reasoning. It's very hard to envisage how that property—whatever it is—being compatible with knowledge in the borderline area. So, knowledge in the borderline area is ruled out. The result will be that in order to address the Observational Paradox, the Traditional Conception of observational properties will have to be given up. So, non-transitive logics are in the same boat as every other theory of vagueness.

Objection Nine: The Maxim of Minimal Mutilation has not played much role in the discussion so far. But surely it should play an important role. For, if we employ it then surely we have a reason to prefer those substantial theories of vagueness which preserve classical logic over those which don't—on the grounds that while both give up on the Traditional Conception of observational predicates, the latter theories mutilate more than the former because they give up on classical logic. So, if we then remain neutral as to which of these remaining classical theories is correct, our Minimal treatment of the Observational Sorites can respect classical logic too. But then Minimalism is mis-advertised as it is no longer fully theory-neutral after all; and indeed despite its lightweight resources it is able to knock non-classical theories out of the running from the outset.

Reply: That's a bit too quick. There are all sorts of different considerations which go into working out whether one theory mutilates more than another. For example, there are various tolerance intuitions which need to be weighed and considered. Moreover, we have only been considering the Observational Sorites Paradox, there are all the other forms of

the Sorites to be entered into the equation (and perhaps other kinds of paradox too). Such a process of weighing and weighting intuitions cannot properly be done from a minimal standpoint.

Objection Ten: Minimalism, as stated, is explicitly neutral as to whether or not classical logic must be given up. But then surely one must be neutral as to whether or not to reason classically in the face of vagueness. Such neutrality must involve not believing that classical logic is valid and not believing that is not valid—a kind of agnosticism. But then such agnosticism mandates a kind of inferential paralysis! A subject will not know whether to reason, e.g., using modus ponens (when vagueness is at large).

Reply One: One response to this worry is simply to drop the strict idea of theory-neutrality and ensure that Minimalism is tied to some particular logical theory. But which logical theory? The previous objection might suggest that Minimalism is best tied to classical logic. But another suggestion is that minimalism must be tied to the logical theory which is the intersection of the various competing logics on offer. On this latter option, Minimalism remains neutral in the sense that no substantial theory contests the rules Minimalism upholds as valid; however, it fails to be theory-neutral in the sense that the minimalist logic of vagueness is weaker than all the other non-minimal logical theories on offer. The result would be a very weak logical theory with little inferential power—a cure which is surely worse than the disease.

Reply Two: A far better reply is to retain the theory-neutrality of Minimalism but resist what are taken to be the consequences of that commitment. So, strictly speaking, one should remain neutral as to which logic is the correct logic for vagueness in administering a minimal treatment of the Observational Sorites. Should that entail a kind of inferential paralysis? No. Recall that the Observational Sorites Paradox took hold given the following two assumptions: that classical logic is valid despite vagueness; and, there are observational predicates—as traditionally conceived. Our first duty as medical practitioners is to treat the disease. We did that by noticing that, whatever the underlying

nature of the disorder, whatever the underlying nature of vagueness, it is cheap, safe, and effective to treat the Observational Sorites by ridding ourselves and others of a problematic belief: the belief that there are observational predicates (as traditionally conceived).

Furthermore, even though we may remain *theoretically* neutral as to whether or not classical logic is valid, we need not be *dialectically* neutral. That is, if we are given no sufficient reason to give up on classical logic then, as things stand in the dialectical situation, we can continue to reason classically. The immediate side-effect of such a remedy is that we are not (yet) in possession of a sufficient *philosophical* justification for accepting classical logic (or any other logic) when confronted with the Observational Sorites Paradox. But we can arguably live with that, given that the minimal treatment has prevented the Paradox from taking hold.⁹⁶

Objection Eleven: But wouldn't the resultant form of Minimalism be a kind of epistemicism in the end? After all, it would involve some kind of commitment to a sharp and unknowable cut-off (in the observational sorites series)?

Reply: It would indeed involve a commitment to a sharp and unknowable cut-off across such a series, but that is not a sufficient condition for an epistemicist conception of the vagueness of observational predicates (minimally conceived). There are various non-epistemicist conceptions of vagueness which employ classical logic and which also take the cut-off drawn by vague predicates (and indeed the truth-values of vague predications) to be unknowable.⁹⁷ The characteristic symptom of an epistemicist conception of vagueness—as mentioned above—is that the explanation as to why we are ignorant

⁹⁶ A minimal remedy may also invoke what Williamson takes to be the strongest argument in favour of classical logic and classical semantics, namely that they “are vastly superior to the alternatives in simplicity, power, past success, and integration with theories in other domains. It would not be wholly unreasonable to insist on these grounds alone that bivalence *must* somehow apply to vague utterances [...]”. But this argument (as it stands) merely establishes that possession of these theoretical virtues provides a strong dialectical reason not to give up on classical logic, rather than a philosophical justification which yields knowledge that classical logic is valid in the presence of vagueness.

⁹⁷ See McGee and McLaughlin (1995), Sorensen (2001), Graff-Fara (2001).

invokes only epistemological resources—such as an account of the nature and source of our limited powers of discrimination. Since a minimal theory does not invoke any epistemological theory as to why we are ignorant—since it does not invoke any substantial theory at all—it does not count as an epistemicist theory of vagueness. We just need to add it to the (growing) list of those theories which are committed to a sharp and unknowable cut-off, but which fall well short of being an epistemic conception of vagueness.

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